Practice Question Lecture # 5

Question:

Find a vector equation of the plane whose parametric equations are given below**:**
$x = 1 - 2t_1 + 3t_2$, $y = 4 - 5t_1 + 6t_2$, $z = 7 - 8t_1 - 9t_2$

Solution:

Since $x = 1 - 2t_1 + 3t_2$, $y = 4 - 5t_1 + 6t_2$, $z = 7 - 8t_1 - 9t_2$
To find the vector equation of the plane, we have to rewrite the three equations as the single vector equation as following:

$$(x, y, z) = (1 - 2t_1 + 3t_2, 4 - 5t_1 + 6t_2, 7 - 8t_1 - 9t_2)$$
$$\Rightarrow \quad = (1, 4, 7) + (-2t_1, -5t_1, -8t_1) + (3t_2, 6t_2, -9t_2)$$
$$\Rightarrow \quad = (1, 4, 7) + t_1(-2, -5, -8) + t_2(3, 6, -9)$$

This is the required equation of the plane that passes through the point $(1, 4, 7)$.

Question:

Find a vector equation of the line in R^2 that passes through the point (1, 3) and is parallel to the vector $\vec{v} = (3, 4)$

Solution:

Let $\overrightarrow{x} = (x, y)$ and $x_0 = (1, 3)$, the vector equation of the line is determined as**:**

$$\overrightarrow{x} = x_0 + \overrightarrow{v}\,t$$
$$(x, y) = (1, 3) + (3, 4)t$$
$$(x, y) = \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} t$$

Question:

Write the vector $\vec{a} = (2,3)$ as a linear combination of the vectors $(1,0)$ and $(0,1)$.

Solution:

To write the vector $\vec{a} = (2,3)$ as a linear combination of the vectors $(1,0)$ and $(0,1)$, we need two scalars

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Question:

If $a_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$, $a_2 = \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$. Determine whether b can be generated as a linear combination of a_1 and a_2?

Solution:

First we see the equation $x_1a_1 + x_2a_2 = b$ has a solution.
To answer this, reduce the augmented matrix $\begin{bmatrix} a_1 & a_2 & b \end{bmatrix}$ in echelon form:

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & -5 & -4 \\ -3 & -2 & 6 \end{bmatrix}$$

$R_2 - 2R_1, R_3 + 3R_1$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -7 & -10 \\ 0 & 1 & 15 \end{bmatrix}$$

$-1/7R_2$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 10/7 \\ 0 & 1 & 15 \end{bmatrix}$$

$R_1 - R_2, R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 11/7 \\ 0 & 1 & 10/7 \\ 0 & 0 & 95/7 \end{bmatrix}$$

We can write this system as**:**

$$x_1 = 11/7$$
$$x_2 = 10/7$$
$$0.x_1 + 0.x_2 = 95/7$$

Which cannot be true for any value of $x_1, x_2 \in \mathbb{R}$.

$\Rightarrow$ Given system has no solution.

$\therefore b \notin Span\{a_1, a_2\}$ i.e. vector b does not lie in the plane spanned by vectors a_1 and a_2.

$\Rightarrow$ b Cannot be generated as a linear combination of a_1 and a_2.

Question:

If $\vec{s} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$ and $\vec{t} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$. Determine whether $\vec{b} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$ is in $Span\{\vec{s}, \vec{t}\}$ or not?

Note: Show the complete steps.

Solution:

To show that $\vec{b}$ is in $Span\{\vec{s}, t\}$, we have to show that $\vec{b}$ can be written as a linear combination of $\vec{s}$ and $\vec{t}$. For this the linear system with augmented matrix $\begin{bmatrix} \vec{s} & \vec{t} & \vec{b} \end{bmatrix}$ should be consistent.

So,

$$\begin{bmatrix} 2 & 1 & 5 \\ 8 & 4 & 15 \end{bmatrix}$$

$1/2R_1$

$$\begin{bmatrix} 1 & 1/2 & 5/2 \\ 8 & 4 & 15 \end{bmatrix}$$

$R_2 - 8R_1$

$$\begin{bmatrix} 1 & 1/2 & 5/2 \\ 0 & 0 & -5 \end{bmatrix}$$

The linear system is not consistent because $0 = -5$ is never true. So, $\vec{b}$ can not be written as a linear combination of $\vec{s}$ and $\vec{t}$. Therefore $\vec{b} \notin Span\{\vec{s}, \vec{t}\}$.