Practice Question with Answer for Lecture # 2

Question # 1

Find the transpose of matrix $A = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$?

Solution:

$$A^t = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$$

Interchange the rows with columns.

Question # 2

What is the order of given matrix $\begin{bmatrix} 5 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$?

Solution: Order is 2×4

Question # 3

Write the following single column matrix as the sum of three column vectors**:**

$$\begin{pmatrix} x^2 + x \\ 3x+1 \\ 9x^2 + e^t \end{pmatrix}$$

Solution:

$$\begin{pmatrix} x^2 + x \\ 3x+1 \\ 9x^2 + e^t \end{pmatrix}\begin{pmatrix} x^2 \\ 0 \\ 9x^2 \end{pmatrix} + \begin{pmatrix} x \\ 3x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ e^t \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 9 \end{pmatrix} x^2 + \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ e^t \end{pmatrix}$$

Question # 4

Find the derivative of the matrix $X(t)=\begin{pmatrix} t^2 \\ \sin t \end{pmatrix}$.

Solution:

$$X'(t)=\begin{pmatrix} 2t \\ \cos t \end{pmatrix}$$

Question # 5

Find, if possible, the multiplicative inverse of the matrix $A=\begin{pmatrix} 3 & 4 \\ 1 & 7 \end{pmatrix}$

Solution:

$$|A|=\begin{vmatrix} 3 & 4 \\ 1 & 7 \end{vmatrix}=21-4=17\neq 0$$

So inverse of A exists.

$$A^{-1}=\frac{1}{|A|}Adj(A)$$

$$Adj(A)=\begin{pmatrix} 7 & -4 \\ -1 & 3 \end{pmatrix}$$

$$A^{-1}=\frac{1}{|A|}Adj(A)=\frac{\begin{pmatrix} 7 & -4 \\ -1 & 3 \end{pmatrix}}{17}=\begin{pmatrix} \frac{7}{17} & \frac{-4}{17} \\ \frac{-1}{17} & \frac{3}{17} \end{pmatrix}$$

Question # 6

Find, if possible, the multiplicative inverse of the given matrices

1. $A=\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{pmatrix}$

Solution:

Solution Since $\det(A)=\begin{vmatrix}1&2&3\\0&4&5\\1&0&6\end{vmatrix}=1(24-0)-2(0-5)+3(0-4)=24+10-12=22\neq 0$

Therefore, the given matrix is non-singular. So, the multiplicative inverse A^{-1} of the matrix A exists. The cofactors corresponding to the entries in each row are

$$C_{11}=\begin{vmatrix}4&5\\0&6\end{vmatrix}=24,\qquad C_{12}=-\begin{vmatrix}2&3\\0&6\end{vmatrix}=-12,\qquad C_{13}=\begin{vmatrix}2&3\\4&5\end{vmatrix}=-2$$

$$C_{21}=-\begin{vmatrix}0&5\\1&6\end{vmatrix}=5,\qquad C_{22}=\begin{vmatrix}1&3\\1&6\end{vmatrix}=3,\qquad C_{23}=-\begin{vmatrix}1&3\\0&5\end{vmatrix}=-5$$

$$C_{31}=\begin{vmatrix}0&4\\1&0\end{vmatrix}=-4,\qquad C_{32}=-\begin{vmatrix}1&2\\1&0\end{vmatrix}=2,\qquad C_{33}=\begin{vmatrix}1&2\\0&4\end{vmatrix}=4$$

Hence $$A^{-1}=\frac{1}{22}\begin{pmatrix}24&-12&-2\\5&3&-5\\-4&2&4\end{pmatrix}=\begin{pmatrix}\frac{24}{22}&\frac{-12}{22}&\frac{-2}{22}\\\frac{5}{22}&\frac{3}{22}&\frac{-5}{22}\\\frac{-4}{22}&\frac{2}{22}&\frac{4}{22}\end{pmatrix}=\begin{pmatrix}\frac{12}{11}&\frac{-6}{11}&\frac{-1}{11}\\\frac{5}{22}&\frac{3}{22}&\frac{-5}{22}\\\frac{-2}{11}&\frac{1}{11}&\frac{2}{11}\end{pmatrix}$$

2. $A=\begin{pmatrix}1&2&3\\4&5&6\\5&7&9\end{pmatrix}$

Solution

Since $\det(A)=\begin{vmatrix}1&2&3\\4&5&6\\5&7&9\end{vmatrix}=1(45-42)-2(36-30)+3(28-25)=3-12+9=0$

Hence inverse does not exist.

Question # 7

For the matrices $A=\begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 0 & 1 \\ 4 & 1 \end{pmatrix}$, evaluate *BA*?

Solution:

$$BA=\begin{pmatrix} 0 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix}=\begin{pmatrix} 0\times 3+1\times 1 & 0\times 5+1\times 1 \\ 4\times 3+1\times 1 & 4\times 5+1\times 1 \end{pmatrix}=\begin{pmatrix} 1 & 1 \\ 13 & 21 \end{pmatrix}$$

Question # 8

For the matrices $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$, evaluate *AB*?

Solution:

$$AB=\begin{pmatrix} 4 & 6 \\ 10 & 15 \\ 16 & 24 \end{pmatrix}$$

Question # 9

For the matrices $A=\begin{pmatrix} 1 & 6 \\ 3 & 7 \\ 5 & 9 \end{pmatrix}$ and $B=\begin{pmatrix} 5 & 0 \\ 3 & 0 \\ 1 & 0 \end{pmatrix}$, evaluate *AB*?

Solution:

Multiplication is not possible as number of columns of first matrix is not equal to the number of rows of second matrix.