Practice Questions Lecture # 3

Question # 1

Which of the following is a non-linear equation?

a. $x_1 + x_2 + x_3 = 0$
b. $x_1 + \sqrt{3}\, x_2 - x_3 = 1$
c. $4x_1x_2 = 1$
d. $x_1 = x_2$

Solution:

$4x_1x_2 = 1$ is non-linear equation because there is product of variables involved. So part c is the correct option.

Question # 2

Which of the following is the correct Coefficient matrix for the system $\begin{matrix} x_1 - x_2 - x_3 = 6 \\ x_1 + 2x_2 = 8 \\ 9x_3 = 1 \end{matrix}$?

a. $\begin{bmatrix} 1 & -1 & 6 \\ 1 & 2 & 8 \\ 0 & 0 & 1 \end{bmatrix}$
b. $\begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 9 \end{bmatrix}$
c. $\begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 8 \\ 9 & 0 & 1 \end{bmatrix}$
d. $\begin{bmatrix} 1 & -1 & -1 & 6 \\ 1 & 2 & 0 & 8 \\ 0 & 0 & 9 & 1 \end{bmatrix}$

Solution:

Part b is the correct option.

Question # 3. Write a system of linear equations corresponding to the following augmented matrices

a. $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 9 & 2 & 6 \\ 6 & 0 & 5 & 3 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix} \qquad \begin{aligned} x_1 &= 2 \\ x_2 &= 1 \\ x_3 &= 4 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 9 & 2 & 6 \\ 6 & 0 & 5 & 3 \end{bmatrix} \qquad \begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ 4x_1 + 9x_2 + 2x_3 &= 6 \\ 6x_1 + 5x_3 &= 3 \end{aligned}$$

Question # 4

Evaluate whether the following system is consistent or inconsistent.

a. $\begin{aligned} 2x + y &= 1 \\ -x - 7y &= 3 \end{aligned}$

b. $\begin{aligned} 2x + 2y &= 5 \\ -2x - 2y &= 3 \end{aligned}$

Solution:

a.

Since $\begin{aligned} 2x + y &= 1 \\ -x - 7y &= 3 \end{aligned}$

The augmented matrix of the system is

$$\begin{pmatrix} 2 & 1 & 1 \\ -1 & -7 & 3 \end{pmatrix}$$

Using row operation, it becomes

$\begin{pmatrix} 1 & -6 & 4 \\ -1 & -7 & 3 \end{pmatrix}$ using $R_1 + R_2$

$\begin{pmatrix} 1 & -6 & 4 \\ 0 & -13 & 7 \end{pmatrix}$ using $R_2 + R_1$

$\begin{pmatrix} 1 & -6 & 4 \\ 0 & 1 & -\frac{7}{13} \end{pmatrix}$ using $-\frac{1}{13}R_2$

$\begin{pmatrix} 1 & 0 & \frac{10}{13} \\ 0 & 1 & -\frac{7}{13} \end{pmatrix}$ using $R_1 + 6R_2$

This corresponds to the system:

$$x = 10/13$$
$$y = -7/13$$

Hence system is consistent.

b.

$$\begin{aligned} 2x + 2y &= 5 \\ -2x - 2y &= 3 \end{aligned}$$

The augmented matrix of the system is

$$\begin{pmatrix} 2 & 2 & 5 \\ -2 & -2 & 3 \end{pmatrix}$$

Using row operation, it becomes

$$\begin{pmatrix} 1 & 1 & 5/2 \\ -2 & -2 & 3 \end{pmatrix} \text{ using } 1/2R_1$$

$$\begin{pmatrix} 1 & 1 & 5/2 \\ 0 & 0 & 8 \end{pmatrix} \text{ using } R_2 + 2R_1$$

This corresponds to the system:

$$x + y = 5/2$$
$$0 = 8$$

Since $0 = 8$ is an mathematically incorrect, the system has no solution. Therefore it is an inconsistent system.

Question # 5

Is $x = 3,\ y = 1$ a solution of the following linear system? If not then give the reason.

$$x - 3y = 1$$
$$5x - 2y = 3$$

Solution:

Put the given values of x and y in the above equation:

$$3 - 3(1) = 0 \neq 1$$
$$5(3) - 2(1) = 13 \neq 3$$

Which shows that these values do not satisfies the above equations so it cannot be solution of the given linear system.

Question # 6

Determine whether the following system is consistent or inconsistent?

$$2x + 3y + 4z = 4$$
$$4x + 5y = 6$$
$$2y + 7z = 1$$

Solution:

$$\begin{aligned} & 2x+3y+4z=4 \\ \text{Since} \quad & 4x+5y=6 \\ & 2y+7z=1 \end{aligned}$$

The augmented matrix of the system is

$$\begin{pmatrix} 2 & 3 & 4 & 4 \\ 4 & 5 & 0 & 6 \\ 0 & 2 & 7 & 1 \end{pmatrix}$$

After applying row operations we get

$$\begin{pmatrix} 1 & 0 & 0 & 7/3 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 1/3 \end{pmatrix}$$

This corresponds to the system

$$x = 7/3$$
$$y = -2/3$$
$$z = 1/3$$

This shows that system is consistent.

Question # 7

Determine whether the following system is consistent or inconsistent?

$$\begin{aligned} & 3x-2y+3z=8 \\ & x+3y+6y=-3 \\ & 2x+6y+12z=-6 \end{aligned}$$

Solution:

Solution:

$$3x-2y+3z=8$$

Since $x+3y+6y=-3$

$$2x+6y+12z=-6$$

The augmented matrix of the system is

$$\begin{pmatrix} 3 & -2 & 3 & 8 \\ 1 & 3 & 6 & -3 \\ 2 & 6 & 12 & -6 \end{pmatrix}$$

After applying row operations we get

$$\begin{pmatrix} 1 & 0 & 21/11 & 18/11 \\ 0 & 1 & 15/11 & -17/11 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

As the last row is all zero which means that two of the planes formed by the equations in the system of equations are parallel, and thus the system of equations is said to have an infinite set of solutions.