LECTURE # 1

Course Objective:

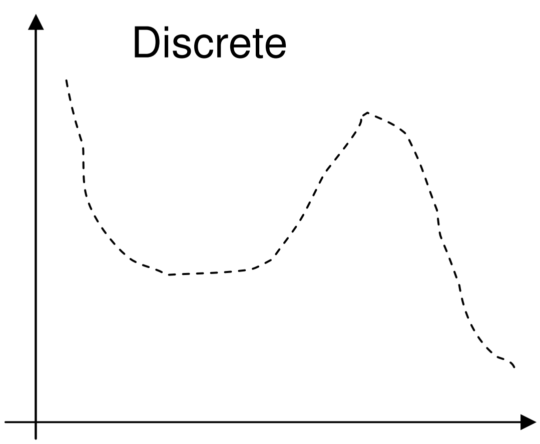
1.Express statements with the precision of formal logic
2.Analyze arguments to test their validity
3.Apply the basic properties and operations related to sets
4.Apply to sets the basic properties and operations related to relations and functions
5.Define terms recursively
6.Prove a formula using mathematical induction
7.Prove statements using direct and indirect methods
8.Compute probability of simple and conditional events
9.Identify and use the formulas of combinatorics in different problems
10.Illustrate the basic definitions of graph theory and properties of graphs
11.Relate each major topic in Discrete Mathematics to an application area in computing

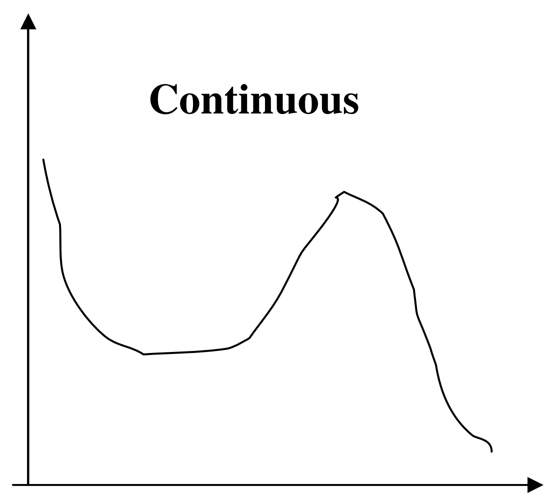
1.Recommended Books:

1.Discrete Mathematics with Applications (second edition) by Susanna S. Epp
2.Discrete Mathematics and Its Applications (fourth edition) by Kenneth H. Rosen
1.Discrete Mathematics by Ross and Wright

MAIN TOPICS:

1. Logic
2. Sets & Operations on sets
3. Relations & Their Properties
4. Functions
5. Sequences & Series
6. Recurrence Relations
7. Mathematical Induction
8. Loop Invariants
9. Loop Invariants
10. Combinatorics
11. Probability
12. Graphs and Trees

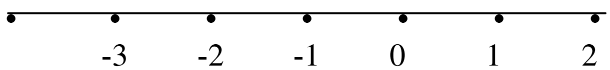




Set of Integers:

• • • • • •
3 -2 -1 0 1 2

Set of Real Numbers:



What is Discrete Mathematics?:

Discrete Mathematics concerns processes that consist of a sequence of individual steps.

LOGIC:

Logic is the study of the principles and methods that distinguishes between a valid and an invalid argument.

SIMPLE STATEMENT:

A statement is a declarative sentence that is either true or false but not both.
A statement is also referred to as a **proposition**
Example: 2+2 = 4, It is Sunday today
If a proposition is true, we say that it has a **truth value** of "**true**".
If a proposition is false, its **truth value** is "**false**".
The truth values **"true"** and **"false"** are, respectively, denoted by the letters **T** and **F**.

EXAMPLES:

1.Grass is green.
2.4 + 2 = 6
2.4 + 2 = 7
3.There are four fingers in a hand.
are propositions

Not Propisitions

- Close the door.
- x is greater than 2.
- He is very rich

are not propositions.

Rule:

If the sentence is preceded by other sentences that make the pronoun or variable reference clear, then the sentence is a statement.

Example:

$x = 1$
$x > 2$
$x > 2$ is a statement with truth-value FALSE.

Example

Bill Gates is an American
He is very rich
He is very rich is a statement with truth-value TRUE.

UNDERSTANDING STATEMENTS:

1.$x + 2$ is positive.	Not a statement
2.May I come in?	Not a statement
3.Logic is interesting.	A statement
4.It is hot today.	A statement
5.$-1 > 0$	A statement
6.$x + y = 12$	Not a statement

COMPOUND STATEMENT:

Simple statements could be used to build a compound statement.

EXAMPLES: **LOGICAL CONNECTIVES**

1. "3 + 2 = 5" **and** "Lahore is a city in Pakistan"
2. "The grass is green" or " It is hot today"
3. "Discrete Mathematics is **not** difficult to me"

AND, OR, NOT are called LOGICAL CONNECTIVES.

SYMBOLIC REPRESENTATION:

Statements are symbolically represented by letters such as ***p, q, r,...***

EXAMPLES:

p **=** "Islamabad is the capital of Pakistan"
q **=** "17 is divisible by 3"

CONNECTIVE	MEANING	SYMBOL	CALLED
Negation	not	~	Tilde
Conjunction	and	$\wedge$	Hat
Disjunction	or	$\vee$	Vel
Conditional	if…then…	$\rightarrow$	Arrow
Biconditional	if and only if	$\leftrightarrow$	Double arrow

EXAMPLES:
p **=** "Islamabad is the capital of Pakistan"
q **=** "17 is divisible by 3"
$p \wedge q$ **=** "Islamabad is the capital of Pakistan and 17 is divisible by 3"
$p \vee q$ **=** "Islamabad is the capital of Pakistan or 17 is divisible by 3"
$\sim p$ **=** "It is not the case that Islamabad is the capital of Pakistan" or simply "Islamabad is not the capital of Pakistan"

TRANSLATING FROM ENGLISH TO SYMBOLS:
Let p = "It is hot", and q = " It is sunny"

SENTENCE	SYMBOLIC FORM
1.It is **not** hot.	$\sim p$
2.It is hot **and** sunny.	$p \wedge q$
3.It is hot **or** sunny.	$p \vee q$
4.It is **not** hot **but** sunny.	$\sim p \wedge q$
5.It is **neither** hot **nor** sunny.	$\sim p \wedge \sim q$

EXAMPLE:
Let ***h*** = "Zia is healthy"
w = "Zia is wealthy"
s = "Zia is wise"

Translate the compound statements to symbolic form:

1.Zia is healthy and wealthy but not wise.	$(h \wedge w) \wedge (\sim s)$
2.Zia is not wealthy but he is healthy and wise.	$\sim w \wedge (h \wedge s)$
3.Zia is neither healthy, wealthy nor wise.	$\sim h \wedge \sim w \wedge \sim s$

TRANSLATING FROM SYMBOLS TO ENGLISH:

Let m = "Ali is good in Mathematics"
  c = "Ali is a Computer Science student"

Translate the following statement forms into plain English:

1. $\sim c$ Ali is not a Computer Science student
2. $c \vee m$ Ali is a Computer Science student or good in Maths.
3. $m \wedge \sim c$ Ali is good in Maths but not a Computer Science student

A convenient method for analyzing a compound statement is to make a truth table for it.

A **truth table** specifies the truth value of a compound proposition for all possible truth values of its constituent propositions.

NEGATION (~):

If ***p*** is a statement variable, then negation of ***p, "not p"***, is denoted as **"~p"**
It has opposite truth value from p i.e.,
if p is true, ~p is false; if p is false, ~p is true.

TRUTH TABLE FOR
~ p

p	*~ p*
T	F
F	T

CONJUNCTION (∧):

If ***p*** and ***q*** are statements, then the conjunction of ***p*** and ***q*** is ***"p and q",*** denoted as ***"$p \wedge q$".***
It is true when, and only when, both p and q are true. If either p or q is false, or if both are false, $p \wedge q$ is false.

TRUTH TABLE FOR
$p \wedge q$

p	*q*	*$p \wedge q$*
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION (∨)
or INCLUSIVE OR

If ***p*** & ***q*** are statements, then the disjunction of ***p*** and ***q*** is ***"p or q",*** denoted as ***"p ∨ q".*** It is true when at least one of p or q is true and is false only when both p and q are false.

TRUTH TABLE FOR
$p \vee q$

p	***q***	***$p \vee q$***
T	T	T
T	F	T
F	T	T
F	F	F